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Two-loop renormalization group restrictions on the standard model and the fourth chiral family

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Abstract. In the framework of the two-loop renormalization group, the restrictions on the Higgs mass from the electroweak vacuum stability and from the absence of the strong coupling are refined, while the more precise value of the top mass is taken into account. When the SM cutoff is equal to the Planck scale, the Higgs mass must be $M_{\rm H} = (161.3 \pm 20.6)^{+4}_{-10}$ GeV and $M_{\rm H} \geq 140.7^{+10}_{-10}$ GeV, where the $M_{\rm H}$ corridor is the theoretical one and the errors are due to the top-mass uncertainty. The SM two-loop β functions are generalized to the case with massive neutrinos from extra families. The requirement of self-consistency of the perturbative SM as an underlying theory up to the Planck scale excludes a fourth chiral family. Under the precision-experiment restriction $M_{\rm H} \leq 215$ GeV, the fourth chiral family, if alone, is excluded even when the SM is regarded as an effective theory. Nevertheless a pair of chiral families constituting a vector-like one could exist.

1 Introduction

The renormalization group (RG) study of a field theory (for a review see, e.g., [1,2]) enables one to grossly understand the structure of the theory as a function of a characteristic energy scale. Of special interest are the cases where the self-consistency of the theory is in danger of being violated. They may signal either a breakdown of the perturbative validity or/and the onset of "new physics".

There are two problems of this kind in the standard model (SM). The first is encountered when some of the running couplings tend to blow up at finite scales. Well-known examples are, e.g., the Landau singularity in QED (more generally, in any Abelian gauge theory U(1)) or in the ϕ^4 scalar theory. In the latter case, the problem is known for a long time as the triviality problem (for a review of triviality arguments see, e.g., [3] and references therein). Technically, we can hoped to solve it by an improvement of the perturbative series or by the development of strong coupling methods; but more probably it has a physical origin, and it could be solved eventually by a more complete theory which should effectively result in a physical cutoff (for an example see, e.g., [4]). In particular, this problem was invoked to justify technicolor as a substitute for the heavy Higgs boson.

The second problem occurs when a running coupling leaves the physical region at some finite scale. In the SM, this happens when the Higgs quartic effective coupling becomes negative, which indicates the absence of a ground state in quantum theory. It is the so-called electroweak vacuum stability problem (for a review see, e.g., [5]). It is a real problem of quantum field theory because this phenomenon takes place in the realm of the perturbative validity. In the framework of the SM, the light Higgs bosons resulting in the unstable electroweak vacuum should be forbidden. On the other hand, if this does happen some new scalar bosons beyond the SM will be required to stabilize the vacuum. Otherwise the light composite Higgs doublet with the compositeness scale corresponding to the scale of the stability breakdown might be a natural solution.

A SM self-consistency study in the framework of the one-loop RG and restrictions thereof on the SM heavy particles, i.e. the Higgs boson and the top quark, was undertaken in [3,6,7]. A generalization to the two-loop level was given in [8–10]. The one-loop RG restrictions on a new heavy chiral family were studied in [11,12], and those on a vector-like family were investigated in [13].

The aim of our present study is twofold. Firstly, we investigate the two-loop RG global profile of the SM in its parameter space at all conceivable scales. This provides us with the background required for the RG study of the SM extension we are looking for. In passing, we refine the RG restrictions on the Higgs mass in the light of the more accurately known top mass and its uncertainty. Secondly and mainly, we generalize the RG study of the SM extended by the fourth chiral family to the two-loop level, and we refine the one-loop self-consistency restrictions thereof on the Higgs and fourth family masses. This requires in turn a generalization of the SM two-loop β functions to the massive neutrino case, which we present.

2 Standard model

The two-loop β functions for a general gauge theory in the $\overline{\mathrm{MS}}$ renormalization scheme are well known in the literature [14], as is their particular realization for the SM [14–16] (compact summaries for the SM can also be found in [17,18]). They are re-collected in a different form in Appendix A.1, where the explicit Yukawa couplings are retained only for the third family. In what follows we put just the generic structure of the emerging one- and two-loop RG differential equations.

Let g_i, $i = 1, 2, 3$, y_f, λ and v be the SM gauge couplings, the Yukawa couplings for fermions f, the Higgs self-interaction coupling and the vacuum expectation value (VEV), respectively. Then one gets

$$
\begin{aligned}
\frac{1}{g_i^3}\frac{\mathrm{d}g_i}{\mathrm{d}\ln\mu} &= \frac{1}{(4\pi)^2}b_{g_i}^{(1)}(g_{i'}) + \frac{1}{(4\pi)^4}b_{g_i}^{(2)}(g_{i'}, y_{f'}),\\
\frac{1}{y_f}\frac{\mathrm{d}y_f}{\mathrm{d}\ln\mu} &= \frac{1}{(4\pi)^2}b_{y_f}^{(1)}(g_{i'}, y_{f'}) + \frac{1}{(4\pi)^4}b_{y_f}^{(2)}(g_{i'}, y_{f'}, \lambda),\\
\frac{\mathrm{d}\lambda}{\mathrm{d}\ln\mu} &= \frac{1}{(4\pi)^2}b_{\lambda}^{(1)}(g_{i'}, y_{f'}, \lambda) + \frac{1}{(4\pi)^4}b_{\lambda}^{(2)}(g_{i'}, y_{f'}, \lambda),\\
\frac{1}{v}\frac{\mathrm{d}v}{\mathrm{d}\ln\mu} &= \frac{1}{(4\pi)^2}b_{v}^{(1)}(g_{i'}, y_{f'}) + \frac{1}{(4\pi)^4}b_{v}^{(2)}(g_{i'}, y_{f'}, \lambda), \quad (1)
\end{aligned}
$$

where μ is a renormalization scale, say in GeV, $b^{(1)}$ and $b^{(2)}$ are one- and two-loop contributions respectively, while $b_{g_i}^{(1)}$ are in fact constants. $g_{i'}$ and $y_{f'}$ are the sets of all g_i and y_f. For simplicity we here neglected the mixing of the Yukawa couplings and thus the CP violating phase. To state it in other terms, the diagonal real form of the Yukawa matrix $Y_{ff'} = \sqrt{2}y_f\delta_{ff'}$ is implied.

The two- and higher-loop contributions to β functions, including the sign, are known to depend in a multi-coupling theory on the renormalization scheme [2]. Hence the physical meaning of the running couplings becomes ambiguous, and it is impossible to improve the perturbative RG analysis of the SM in a scheme-independent way beyond one loop.

We integrated the RG (1) numerically for $\mu \geq M_{\mathrm{Z}}$ with the following initial conditions at the M_{Z} scale:

$$
\begin{aligned}
\alpha_1(M_{\mathrm{Z}}) &= 0.0102,\\
\alpha_2(M_{\mathrm{Z}}) &= 0.0338,\\
\alpha_3(M_{\mathrm{Z}}) &= 0.123, \quad (2)
\end{aligned}
$$

which is in accordance with $\alpha(M_{\mathrm{Z}}) = 1/127.90$ and $\sin^2\theta_{\mathrm{W}}(M_{\mathrm{Z}}) = 0.2315$ [19]. Our normalizations of the gauge couplings are as follows: $g_1 = (5/3)^{1/2}g'$, $g_2 \equiv g$ and $g_3 \equiv g_{\mathrm{S}}$, with g', g and g_{S} being the conventional SM couplings. We also choose the relations $m_f = y_f v$ and $m_{\mathrm{H}} = \lambda^{1/2}v$ as the definition of normalization for the Higgs and Yukawa couplings, where $v = (\sqrt{2}G_{\mathrm{F}})^{-1/2} = 246.22\,\mathrm{GeV}$ is the Higgs VEV. Because the evolution of $v(\mu)$ is gauge dependent, we use in what follows only the gauge-independent observable $v \equiv v(M_{\mathrm{Z}})$.

Besides, at $\mu = M_{\mathrm{Z}}$ we use the one-loop matching condition for the physical M_f and the running $m_f(\mu) \equiv y_f(\mu)v$ masses of the fermions $f = q$ and l given by

$$
m_f(\mu) = M_f\left(1 + \delta_f^{\mathrm{QCD}}(\mu) + \delta_f^{\mathrm{QED}}(\mu) + \delta_f^{(\mathrm{t,H})}(\mu)\right). \quad (3)
$$

Here one has

$$
\delta_q^{\mathrm{QCD}}(\mu) = -\frac{4}{3}\frac{\alpha_3(\mu)}{\pi}\left(1 + \frac{3}{4}\ln\frac{\mu^2}{M_q^2}\right). \quad (4)
$$

δ_f^{QED} is obtained from the expression by substituting $4/3\alpha_3$ with $Q_f^2\alpha$, where Q_f is the electric charge of the fermion f. The radiative corrections induced by the top quark and the Higgs boson, $\delta_f^{(\mathrm{t,H})}$, can be found in [20]:

$$
\begin{aligned}
\delta_\tau^{(\mathrm{t,H})}(\mu) &= \frac{1}{(4\pi)^2}\left(\frac{M_{\mathrm{t}}}{v}\right)^2\left(3\ln\frac{\mu^2}{M_{\mathrm{t}}^2} + \frac{3}{2} + \frac{1}{4}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{t}}^2}\right),\\
\delta_{\mathrm{b}}^{(\mathrm{t,H})}(\mu) &= \frac{1}{(4\pi)^2}\left(\frac{M_{\mathrm{t}}}{v}\right)^2\left(\frac{3}{2}\ln\frac{\mu^2}{M_{\mathrm{t}}^2} + \frac{1}{4} + \frac{1}{4}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{t}}^2}\right),\\
\delta_{\mathrm{t}}^{(\mathrm{t,H})}(\mu) &= \frac{1}{(4\pi)^2}\left(\frac{M_{\mathrm{t}}}{v}\right)^2\left(\frac{9}{2}\ln\frac{\mu^2}{M_{\mathrm{t}}^2} + \frac{11}{2} - 2\pi\frac{M_{\mathrm{H}}}{M_{\mathrm{t}}}\right),
\end{aligned}
\quad (5)
$$

where the last line is valid for $(M_{\mathrm{H}}/2M_{\mathrm{t}})^2 \ll 1$. Similarly, the initial value $m_{\mathrm{H}}(M_{\mathrm{Z}})$ is related to the physical Higgs mass M_{H} through the running mass $m_{\mathrm{H}}(\mu) \equiv y_{\mathrm{H}}(\mu)v$ at the scale M_{Z}, where

$$
m_{\mathrm{H}}(\mu) = M_{\mathrm{H}}\left(1 + \delta_{\mathrm{H}}(\mu)\right). \quad (6)
$$

In the limit $(M_{\mathrm{H}}/2M_{\mathrm{t}})^2 \ll 1$ one has the following asymptotic one-loop expression [20,21]:

$$
\begin{aligned}
(4\pi)^2\delta_{\mathrm{H}}(\mu) &= \left(\frac{M_{\mathrm{H}}}{v}\right)^2\left(3\ln\frac{\mu^2}{M_{\mathrm{H}}^2} + 6 - \frac{3\sqrt{3}\pi}{4}\right)\\
&+ \left(\frac{M_{\mathrm{t}}}{v}\right)^2\left(3\ln\frac{\mu^2}{M_{\mathrm{t}}^2} - 2 + \frac{3}{10}\frac{M_{\mathrm{H}}^2}{M_{\mathrm{t}}^2}\right)\\
&+ \sum_{V=\mathrm{W}^\pm,\mathrm{Z}}\left(\frac{M_V}{v}\right)^2\left[\frac{3}{2}\ln\frac{M_V^2}{\mu^2}\right.\\
&+\frac{1}{4}\frac{m_{\mathrm{H}}^2}{M_V^2}\ln\frac{M_{\mathrm{H}}^2}{M_V^2} - \frac{5}{2} + 6\frac{M_V^2}{M_{\mathrm{H}}^2}\\
&\left.-2\left(3\frac{M_V^2}{M_{\mathrm{H}}^2} + \frac{1}{4}\frac{M_{\mathrm{H}}^2}{M_V^2} - 1\right)f\left(\frac{M_V^2}{M_{\mathrm{H}}^2}\right)\right], \quad (7)
\end{aligned}
$$

where

$$
f(x) = \begin{cases} (4x-1)^{1/2}\mathrm{arctg}(4x-1)^{-1/2}, & x > \frac{1}{4}\\ \frac{1}{2}(1-4x)^{1/2}\ln\frac{1+(1-4x)^{1/2}}{1-(1-4x)^{1/2}}, & x < \frac{1}{4}. \end{cases} \quad (8)
$$

When we take this all together, we finally get at $M_{\mathrm{H}} = 150\,\mathrm{GeV}$:

$$
\begin{aligned}
m_\tau(M_{\mathrm{Z}}) &= 1.764\,\mathrm{GeV},\\
m_{\mathrm{b}}(M_{\mathrm{Z}}) &= (4.47 \pm 0.50)\,\mathrm{GeV},\\
m_{\mathrm{t}}(M_{\mathrm{Z}}) &= (171.8^{+4.6}_{-4.7})\,\mathrm{GeV}. \quad (9)
\end{aligned}
$$

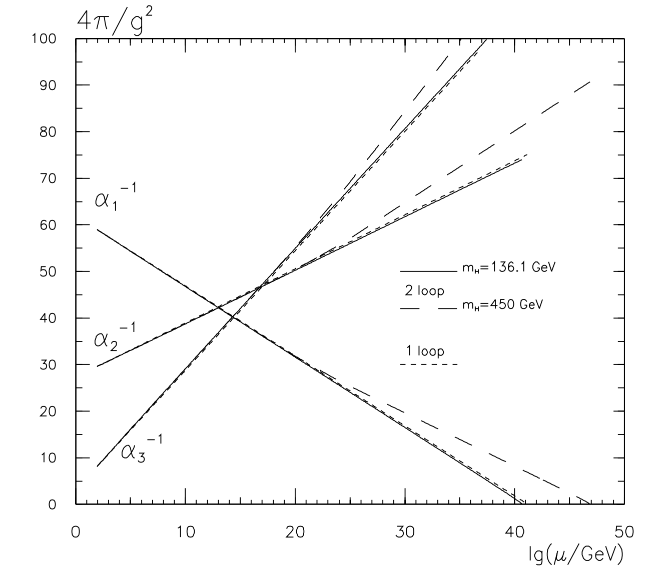

Fig. 1. Running of the inverse gauge couplings squared α_i^{-1}, $i = 1, 2, 3$. The number of generations is $n_\mathrm{g} = 3$. The represented Higgs masses are those corresponding to the typical heavy Higgs and to the lower critical Higgs curve shown in bold in Fig. 3.

The last two values correspond to the physical bottom and top masses $M_\mathrm{b} = (4.5 \pm 0.5)\,\mathrm{GeV}$ [19] and $M_\mathrm{t} = (175 \pm 5)\,\mathrm{GeV}$ [22], respectively. Only errors in the top mass are left as the main source of the subsequent uncertainties.

As a field theory, the SM can legitimately be pulled to its inner ultimate limits. This may help to better understand its structure in the physically reasonable region $\mu < M_\mathrm{Pl}$, $M_\mathrm{Pl} \simeq 10^{19}\,\mathrm{GeV}$, which is to be considered more seriously. So all the subsequent numerical results are obtained at all allowed μ with the exact two-loop β functions. Most of the terms in the latter ones proved to be crucial for the quantitative evolution of couplings in the physical μ region up to the Planck scale. However, for the qualitative analysis of the SM RG solutions at extremely high μ, $\mu \gg M_\mathrm{Pl}$, we only retain the most representative term in the coefficients of the β functions given below.

To estimate the dependence of the results on the loop order and to pick out regions where perturbation theory may be more reliably trusted, we present both the one- and two-loop results. They are shown in Figs. 1–5. Let us discuss them in turn for the gauge, Yukawa and Higgs sectors of the SM.

2.1 Gauge sector

Figure 1 shows the running with μ of the inverse gauge couplings squared. Under the simplifications adopted, one has (with the number of generations here and in what follows $n_\mathrm{g} = 3$)

$$(4\pi)^2 \beta_{g_1}^{(1)} = \frac{41}{10},$$
$$(4\pi)^4 \beta_{g_1}^{(2)} = \frac{199}{50} g_1^2 - \frac{17}{5} y_\mathrm{t}^2 + \cdots . \qquad (10)$$

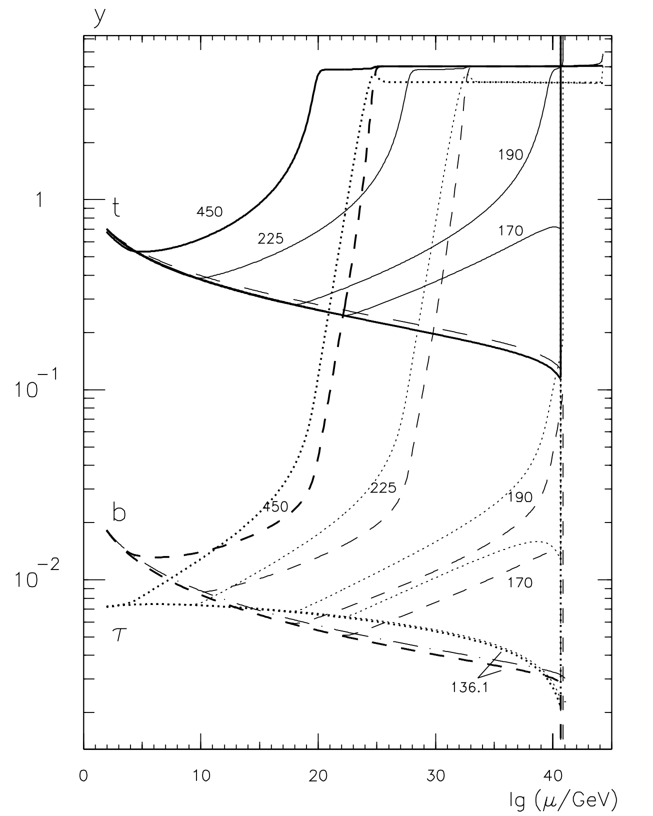

Fig. 2. Running of the third family Yukawa couplings ($n_\mathrm{g} = 3$). The decreasing curves shown in bold correspond to the lower critical Higgs mass. The thin lines, close to the latter bold lines, correspond to the one-loop approximation.

It can be seen that at the one-loop order the coupling g_1 develops a pole singularity at $\Lambda_{g_1}^{(1)}$, $\log \Lambda_{g_1}^{(1)} = 41$. The validity of the perturbation theory in g_1 restricts $\alpha_1 \le 4\pi$ and hence $\log \mu \le 40$, which is in the logarithmic scale twice as large as the Planck scale.

As it is seen from Fig. 1, the actual influence of $y_f(\lambda(\mu))$ on g_1 in two loops is somewhat sizable only for heavy Higgs. It diminishes the slope of $g_1(\mu)$ at μ beyond the Planck scale, where y_f are large, and shifts the singularity position $\Lambda_{g_1}^{(2)}$ upwards to $\log \Lambda_{g_1}^{(2)} = 47$ for the heavy Higgs ($m_\mathrm{H}(M_\mathrm{Z}) = 450\,\mathrm{GeV}$), which is close to that maximally allowed by the perturbative consistency in λ. The value $m_\mathrm{H}(M_\mathrm{Z}) = 136.1\,\mathrm{GeV}$ corresponds to the critical lower bound of the electroweak vacuum stability (see later on). Curves corresponding to the lighter Higgs bosons are very close to that for $m_\mathrm{H}(M_\mathrm{Z}) = 136.1\,\mathrm{GeV}$. Curves for the intermediate values of $m_\mathrm{H}(M_\mathrm{Z})$, i.e. $136.1\,\mathrm{GeV} < m_\mathrm{H}(M_\mathrm{Z}) < 450\,\mathrm{GeV}$, are located in between the two extreme cases.

2.2 Yukawa sector

Figure 2 depicts the evolution of the Yukawa couplings y_f for the third family SM fermions: t and b quarks, and the τ lepton. For the top quark one has approximately

$$(4\pi)^2\beta^{(1)}_{y_t} = 9y_t^2 - \frac{17}{20}g_1^2 - \frac{9}{4}g_2^2 - 8g_3^2 + \cdots,$$
$$(4\pi)^4\beta^{(2)}_{y_t} = -48y_t^4 + \frac{3}{2}\lambda^2 - 12y_t^2\lambda + \cdots. \quad (11)$$

In one loop, $\beta_f^{(1)}$ are dominated by the negative gauge contributions, so that all the y_f are decreasing with μ and lie in the weak coupling regime.[1]

However, in two loops the behavior changes drastically. An approximate UV-stable fixed point appears at $y_t^{(\mathrm{UV})} \simeq 5.4$ due to compensation of the y_t^2 and y_t^4 contributions. In the real world, this critical value is approached from below, both for the t and b quarks and for the τ lepton, the faster the heavier the Higgs boson is. Hence for a sufficiently heavy Higgs, $m_H(M_Z) \geq 215$ GeV, all third family fermions would fall into the strong coupling regime at sufficiently high μ. This would make the third family fermions much more alike at the high scales than at the electroweak scale. In practice, prior to M_{Pl} the strong coupling develops only for the t quark when the Higgs is rather heavy, $m_H(M_Z) \geq 450$ GeV. Because from the combined LEP data on the precision experiments it follows that $M_H \leq 215$ GeV at 95% C.L. [23], one may conclude that the Yukawa sector of the SM is weakly coupled along the whole physically reasonable region of μ, $\mu \leq M_{Pl}$.

2.3 Higgs sector

Figure 3 presents running of the Higgs quartic coupling. For this one approximately has

$$(4\pi)^2\beta^{(1)}_{\lambda} = 12\lambda^2 - 48y_t^4 + \frac{27}{100}g_1^4 + (24y_t^2 - \frac{9}{5}g_1^2)\lambda + \cdots,$$
$$(4\pi)^4\beta^{(2)}_{\lambda} = -78\lambda^3 - 3.411g_1^6 + \cdots. \quad (12)$$

The β function for the pure Higgs sector is known in the $\overline{\mathrm{MS}}$ scheme up to the three-loop order [24]

$$(4\pi)^6\beta^{(3)}_{\lambda} = (897 + 504\zeta(3))\,\lambda^4. \quad (13)$$

It is scheme dependent.

In two loops, there are three critical curves shown in bold. First of all, there appears an approximate UV-stable fixed point at $\lambda^{1/2}_{\mathrm{UV}} \simeq 4.93$ produced by the compensation of the one- and two-loop terms: λ^2 and λ^3. It corresponds to a boundary value of the Higgs mass $m^{(2)}_{\mathrm{H\,max}}(M_Z) = 1200$ GeV, at and above which the theory is definitely strongly coupled. The boundary Higgs mass for the vacuum instability in two loops is $m^{(2)}_{\mathrm{H\,min}}(M_Z) = 136.1$ GeV. The third critical value $m^{(2)}_{\mathrm{H\,inter}}(M_Z) = 156.7$ GeV borders the region with the potentially strongly coupled Higgs from that with the weakly coupled Higgs. Note that theory with $m^{(2)}_{\mathrm{H\,min}} < m_H(M_Z) < m^{(2)}_{\mathrm{H\,inter}}$ is consistent in two loops up to the ultimate scale $\mu = \Lambda^{(2)}_{g_1}$.

[1] To be more precise, the one-loop trajectory for the τ lepton is mildly convex, so that it intersects with the curve for the b quark near the GUT scale.

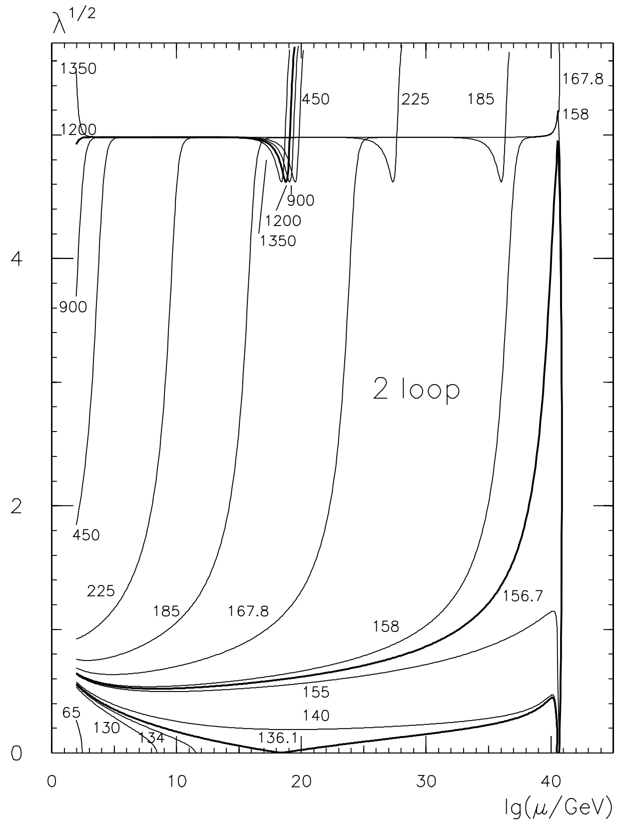

Fig. 3. Running of the Higgs quartic coupling ($n_g = 3$) in two loops. The critical curves are shown in bold.

For completeness, we present in Fig. 4 the plot for $v(\mu)$ in the 't Hooft–Landau gauge both in one and two loops. We see that the electroweak symmetry never restores prior to the Plank scale.

Finally, one can impose the requirement of the SM self-consistency up to some cutoff scale Λ. In other terms, the theory should be neither strongly coupled nor unstable at $\mu \leq \Lambda$. In one loop, this means that the λ singularity position fulfills the requirement $\Lambda^{(1)}_{\lambda} \geq \Lambda$, and simultaneously one has $\mu|_{\lambda=0} \geq \Lambda$. In two loops, we should choose as a criterion for the onset of the strong coupling regime the requirements $\beta^{(2)}_{\lambda}/\beta^{(1)}_{\lambda}|_{\Lambda}$ and $\beta^{(3)}_{\lambda}/\beta^{(2)}_{\lambda}|_{\Lambda} < 1$, which guarantee the perturbativity and reduce the scheme dependence. When we neglect all couplings but λ, this would mean that $\lambda^{1/2} \leq 2$, in particular $m_H(M_Z) \leq 500$ GeV, the restriction we retain for the realistic case. Because we do not know $\beta^{(3)}_t$, we restrict ourselves just to the requirement that $\beta^{(2)}_t/\beta^{(1)}_t|_{\Lambda} < 1$ which is definitely fulfilled at $y_t \leq 2 < y_t^{(\mathrm{UV})}$.

The one- and two-loop restrictions are drawn in Fig. 5. Here use is made of the exact one-loop equation (6) for matching between the $\overline{\mathrm{MS}}$ value $m_H(M_Z)$ and the physical Higgs mass M_H. The sensitivity of the allowed region of the Higgs mass to the uncertainty of the top-quark mass is also indicated. Strictly speaking, the region between the highest and the lowest curves is allowed. This means that for $\Lambda = M_{Pl}$ the legitimate Higgs mass is

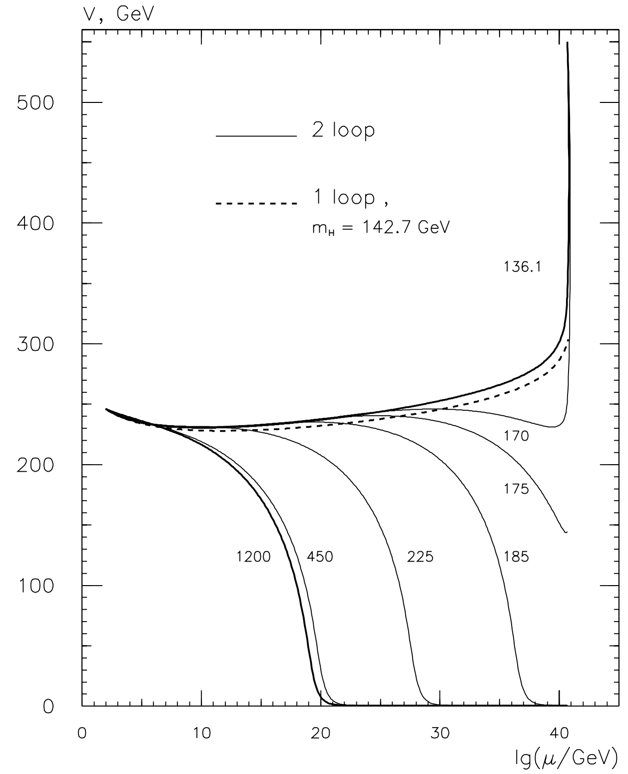

Fig. 4. Running of the Higgs VEV ($n_{\mathrm{g}} = 3$) in the 't Hooft–Landau gauge.

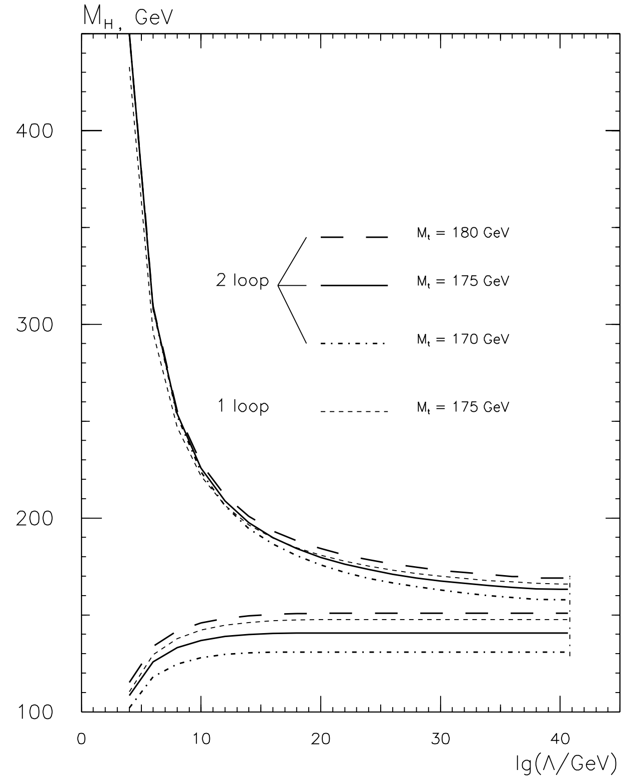

Fig. 5. The SM one- and two-loop self-consistency plot ($n_{\mathrm{g}} = 3$): the allowed Higgs mass vs. the cutoff scale Λ.

$M_{\mathrm{H}} = (161.3 \pm 20.6)^{+4}_{-10}\,\mathrm{GeV}$. One also gets the lower bound $M_{\mathrm{H}} \geq 140.7 \pm 10\,\mathrm{GeV}$ at such a cutoff.[2]

3 The fourth chiral family

Since the two-loop RG global profile of the SM is understood, one is in a position to discuss the SM conceivable extensions. Here we consider the minimum SM extension by means of additional heavy fermion families. If alone, the fourth family should necessarily have the same chirality pattern as the three light families. This must be required to avoid the potential problem of large direct mass mixing for the fourth family with the light ones.

Concerning the fifth family, there are two possibilities: either it has the same chirality as the four previous families, or it is a mirror one (or, to state it differently, it is charge conjugate with respect to the rest of the families). In the first case, the analysis repeats itself with just more parameters. In the second case, which may likewise be attributed to one vector-like family, the large direct mass terms could be introduced for the pair of the fourth and fifth families, in addition to Yukawa couplings. This enormously proliferates the number of free parameters and makes the general analysis complicated. On the other hand, if one adopts a mass-independent renormalization, say $\overline{\mathrm{MS}}$, the net influence of the direct mass terms on the evolution of the SM parameters will be just in the threshold effects. Barring them, this case is technically similar to the case with two extra chiral families.

For these reasons, we restrict ourselves to one new chiral family. In order to conform with the experimental number of light neutrinos ($n_\nu = 3$), we should also add the right-handed neutrinos ν_{R} (at least for the fourth family) and the proper Yukawa couplings for them. The right-handed neutrinos may possess the explicit Majorana mass as well, so that the physical neutrino masses may be quite different from their Yukawa counterparts. Because in the mass-independent renormalization the explicit mass terms are important only in the threshold effects, we disregard them in what follows. We generalized the two-loop SM β functions of [14] to the case with the neutrino Yukawa couplings. The results are given in Appendix A.2. The rest of the β functions is as in [14]. For practical calculations with the fourth family we neglected the light neutrino Yukawa couplings. The reduced four-family β functions are given in Appendix A.3.

At present, there are no theoretical hints on the existence (or the non-existence) of a fourth (and higher) family. Nevertheless, one can extract some restrictions on the corresponding fermion masses. These restrictions are

[2] The true condition for the electroweak vacuum stability is the existence of a global minimum in the Higgs effective potential [25,26]. For the two-loop RG improved one-loop effective potential $V_{\mathrm{eff}}(\mu, \phi)$ this turns out at $\Lambda \simeq M_{\mathrm{Pl}}$ to be in practice equivalent to our requirement that the running coupling does not become negative. At the lower values of Λ there are discrepancies which we attribute to the difference of the stability criteria.

twofold: the direct and indirect ones; these are in a sense complementary to each other. The first group gives bounds on the common mass scale of the fourth family, the second one restricts the splitting of the masses inside the family.

The existing direct experimental bounds on the masses of the fourth family quarks t_4 and b_4 depend somewhat on the assumptions about their decays. If the lightest of the quarks, say b_4, is stable enough to leave the detector, the limit on its mass is $M_4 \geq 140\,\mathrm{GeV}$ [27]. On the other hand, for unstable quarks, decaying inside the detector, the limit can be estimated from the CDF and D0 searches for the top quark [22] to be about M_t. Concerning the neutral and charged leptons of the fourth family, ν_4 and e_4, it follows from LEP searches that $M_{\nu_4} \geq 59\,\mathrm{GeV}$ and $M_{e_4} \geq 90\,\mathrm{GeV}$ at 95% C.L. [19,28].

The indirect restrictions can be extracted from the precision electroweak data, and they are related to the absence of decoupling with respect to the heavy chiral fermions. This results in the quadratically increasing dependence of the electroweak radiative corrections on the heavy fermion masses. To avoid such large corrections, as the precision data require, the masses of a heavy fermion doublet should be highly degenerate. Namely, one should have for the quarks t_4 and b_4 that $(M_{t_4}^2 - M_{b_4}^2)/M_Z^2 \leq 1$; for the leptons ν_4, e_4 it is similar.[3]

To reduce the number of free parameters we assume in what follows that $m_{t_4} = m_{b_4} = m_Q$ and $m_{\nu_4} = m_{e_4} = m_L$. As representative, we considered two cases: $m_L/m_Q = 1/2$ and 1, with the common mass m_Q of the heavy quarks given by the fourth family scale m_4. It follows that both these typical cases do not contradict the direct experimental bounds if $m_4 \geq 180\,\mathrm{GeV}$. Our results for the case $m_4 = 200\,\mathrm{GeV}$, which we consider as a typical one, are presented in Figs. 6–9.

Figure 6 shows the evolution of α_i^{-1} with μ. It is seen that the two-loop contributions manifest themselves at rather low scales, $\mu = 10^7$–$10^8\,\mathrm{GeV}$. They are governed by the onset of the strong coupling regime in the Yukawa sector at such a μ (see Fig. 7). Accordingly, the perturbatively consistent region of μ in the Higgs sector shrinks to the same values (see Fig. 8). When we now apply the same criteria of self-consistency as in the case of the minimal SM, we get the allowed values of M_H, depending on the cutoff scale Λ (Fig. 9). The sensitivity of the restrictions to the shift in the mass m_4 is also indicated. The dependence on ΔM_t is much smaller, and it is not shown.

Finally, Fig. 10 presents the two-loop allowed regions in the m_4–M_H plane under the restriction on the Yukawa couplings $y \leq 1.5$. The direct influence of the Yukawa perturbative validity in two loops on the allowed regions of m_4 and M_H is rather weak at high Λ. The figure excludes the fourth heavy chiral family at high Λ, $\Lambda \geq 10^{10}\,\mathrm{GeV}$, independent of the Higgs mass. Under LEP restriction $M_H \leq 215\,\mathrm{GeV}$ at 95% C.L.; the fourth chiral family is

[3] One important peculiarity of the vector-like family is the decoupling with respect to the explicit mass term when the Yukawa couplings are fixed. Hence, unlike in the case of a chiral family, there is no need here for fine tuning in the Yukawa couplings to suppress the large radiative corrections.

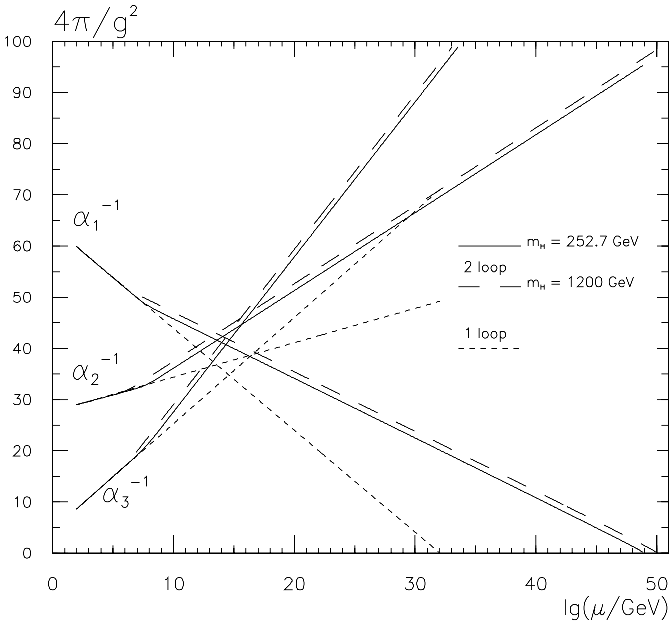

Fig. 6. Running of the gauge couplings ($n_g = 4$). The fourth family mass scale is $m_4 = 200\,\mathrm{GeV}$ and $m_L/m_Q = 1/2$.

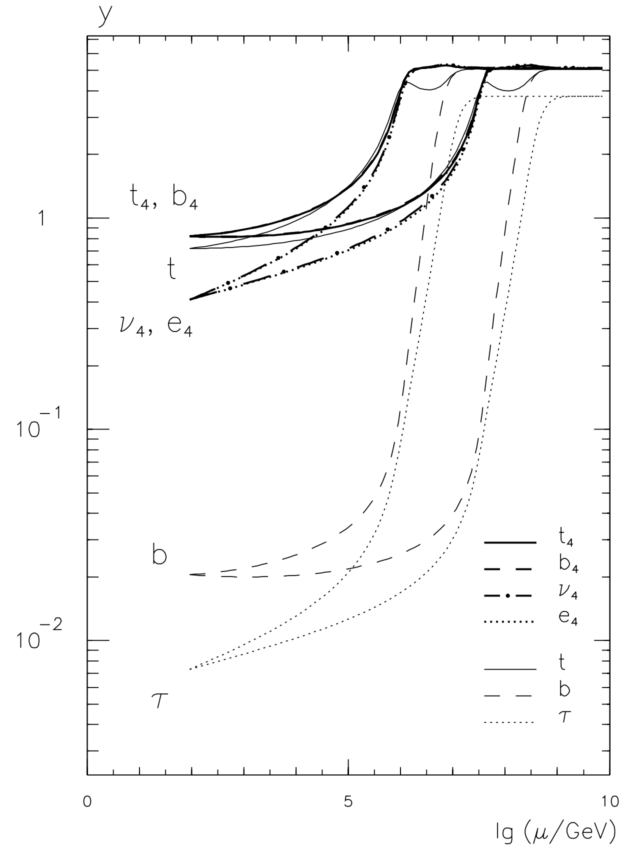

Fig. 7. Two-loop running of the Yukawa couplings ($n_g = 4$) for the third and fourth families at $m_4 = 200\,\mathrm{GeV}$ and $m_L/m_Q = 1/2$. The upper and lower curves correspond to the Higgs masses, respectively, for the upper and lower Higgs critical curves shown in bold in Fig. 8.

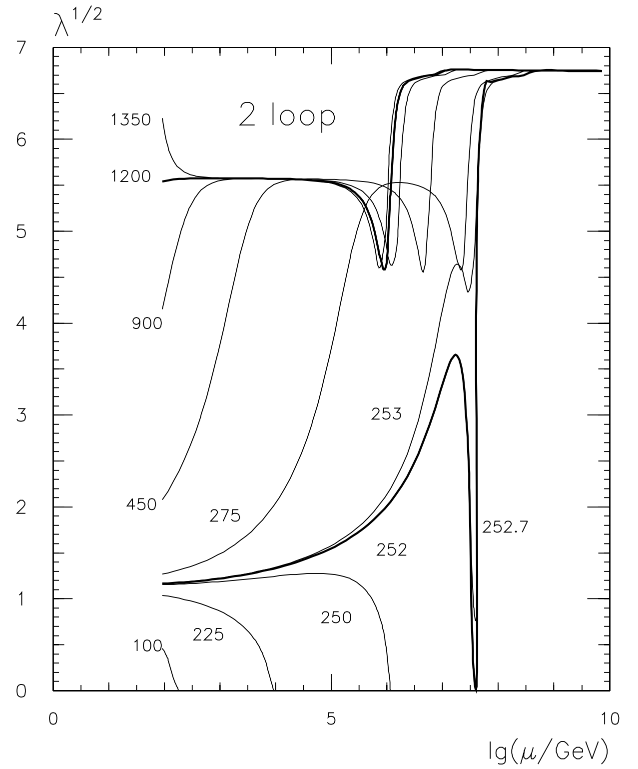

Fig. 8. Two-loop running of the Higgs quartic coupling ($n_\mathrm{g} = 4$) at $m_4 = 200\,\mathrm{GeV}$ and $m_L/m_Q = 1/2$.

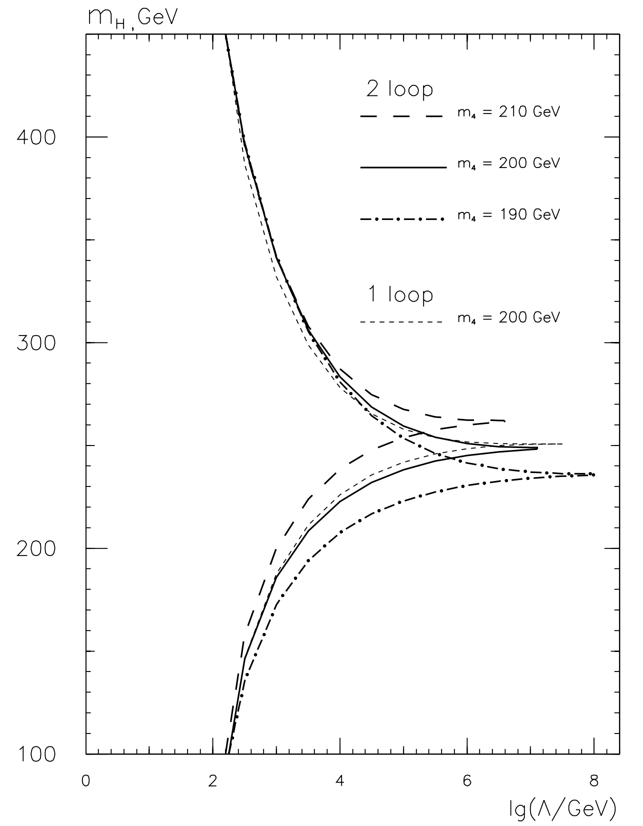

Fig. 9. One- an two-loop self-consistency plot ($n_\mathrm{g} = 4$): the allowed Higgs mass vs. the cutoff scale Λ at $m_4 = 200\,\mathrm{GeV}$ and $m_L/m_Q = 1/2$.

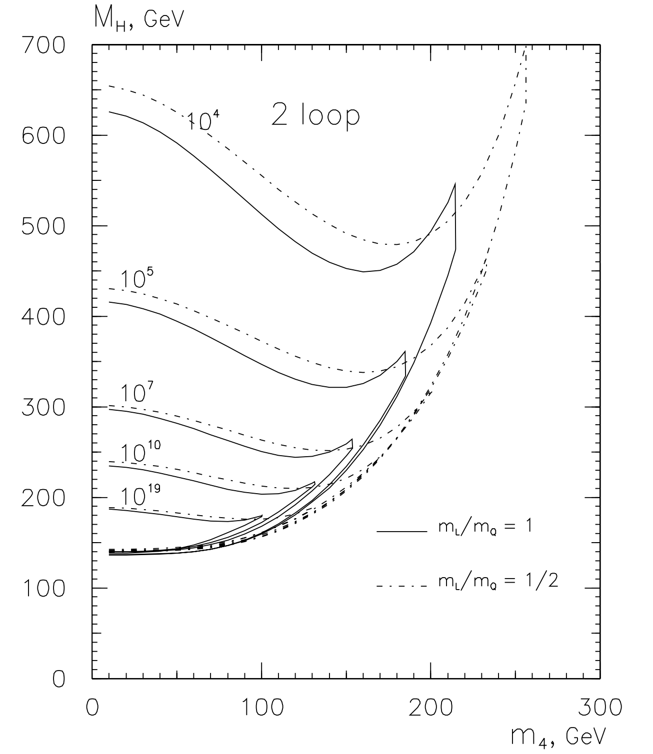

Fig. 10. Two-loop self-consistency plot under the restriction $y \leq 1.5$ on the Yukawa couplings ($n_\mathrm{g} = 4$): the allowed Higgs mass vs. the fourth family scale m_4. The cutoff scale Λ in GeV is fixed and $M_\mathrm{t} = 175\,\mathrm{GeV}$.

completely excluded at this C.L.[4] The dependence of the restrictions on the top mass uncertainty is very faint. The weaker restriction on the Yukawa couplings $y \leq 2$ results in a reduction of the cutoff for $m_4 > 200\,\mathrm{GeV}$ but practically does not influence the restrictions for $\Lambda \geq 10^5\,\mathrm{GeV}$.

4 Conclusions

Let us summarize the differences in the RG global profiles of the SM with three and four chiral families. For three families with the experimentally known masses, the Yukawa sector stays weakly coupled up to the Planck scale for all experimentally preferred values of the Higgs mass, $M_\mathrm{H} \leq 215\,\mathrm{GeV}$. The validity of the perturbative SM up to the Planck scale, including the Yukawa sector, as well as the vacuum stability require the Higgs mass to be $M_\mathrm{H} = (161.3 \pm 20.6)^{+4}_{-10}\,\mathrm{GeV}$ and $M_\mathrm{H} \geq 140.7^{+10}_{-10}\,\mathrm{GeV}$. Here the M_H corridor is the theoretical one; the errors are produced by the top mass uncertainty.

The inclusion of the fourth heavy chiral family qualitatively changes the mode of the SM realization. With the addition of the family, the strong coupling is driven in one

[4] Though a possible caveat emerges if one assumes that the fourth family is vector-like and that, unnatural as it may seem, its Yukawa couplings are small. Then the ensuing restrictions on the family are strongly reduced, and the vector-like fourth family could still exist.

loop by the Yukawa interactions. It also transmits to the Higgs self-interactions at the one-loop order. Hence the strong coupling develops in both these sectors in parallel, and their couplings blow up at sufficiently low scales. As a result, the requirement of self-consistency of the perturbative SM as an underlying theory up to the Planck or GUT scale excludes the fourth chiral family. However, as an effective theory, the SM allows a heavy chiral family with a mass up to 250 GeV, depending on the Higgs mass and the cutoff scale. Under the precision-experiment restriction $M_{\mathrm{H}} \leq 215\,\mathrm{GeV}$, the fourth chiral family, taken alone, is excluded. Nevertheless, a pair of chiral families constituting a vector-like one could still exist.

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Appendices

A.1 SM β functions

Here $n_{\mathrm{g}} = 3$ and the generation index g runs over the values $g = 1, 2, 3$. One has in fact $\sum_g \equiv \delta_{g3}$. Here and in what follows the parts of the expressions for the anomalous dimension γ_v which are proportional to the gauge couplings are valid in the 't Hooft–Landau gauge $\xi = 0$.

One-loop contributions

(i) Gauge sector:

$$(4\pi)^2 g_1^{-3}\beta_{g_1}^{(1)} = \tfrac{41}{10},$$
$$(4\pi)^2 g_2^{-3}\beta_{g_2}^{(1)} = -\tfrac{19}{6},$$
$$(4\pi)^2 g_3^{-3}\beta_{g_3}^{(1)} = -7.$$

(ii) Yukawa sector:

$$(4\pi)^2 y_\tau^{-1}\beta_{y_\tau}^{(1)} = 3y_\tau^2 + 2\textstyle\sum_g(3y_{\mathrm{u}_g}^2 + 3y_{\mathrm{d}_g}^2 + y_{\mathrm{e}_g}^2) - \tfrac{9}{4}g_1^2 - \tfrac{9}{4}g_2^2,$$
$$\begin{aligned}(4\pi)^2 y_{\mathrm{t}}^{-1}\beta_{y_{\mathrm{t}}}^{(1)} = {} & 3y_{\mathrm{t}}^2 - 3y_{\mathrm{b}}^2 + 2\textstyle\sum_g(3y_{\mathrm{u}_g}^2 + 3y_{\mathrm{d}_g}^2 + y_{\mathrm{e}_g}^2)\\ & - \tfrac{17}{20}g_1^2 - \tfrac{9}{4}g_2^2 - 8g_3^2,\end{aligned}$$
$$\begin{aligned}(4\pi)^2 y_{\mathrm{b}}^{-1}\beta_{y_{\mathrm{b}}}^{(1)} = {} & 3y_{\mathrm{b}}^2 - 3y_{\mathrm{t}}^2 + 2\textstyle\sum_g(3y_{\mathrm{u}_g}^2 + 3y_{\mathrm{d}_g}^2 + y_{\mathrm{e}_g}^2)\\ & - \tfrac{1}{4}g_1^2 - \tfrac{9}{4}g_2^2 - 8g_3^2.\end{aligned}$$

(iii) Higgs sector:

$$\begin{aligned}(4\pi)^2\beta_\lambda^{(1)} = {} & 12\lambda^2 + 8\lambda\textstyle\sum_g(3y_{\mathrm{u}_g}^2 + 3y_{\mathrm{d}_g}^2 + y_{\mathrm{e}_g}^2)\\ & - 9\lambda(\tfrac{1}{5}g_1^2 + g_2^2) - 16\textstyle\sum_g(3y_{\mathrm{u}_g}^4 + 3y_{\mathrm{d}_g}^4 + y_{\mathrm{e}_g}^4)\\ & + \tfrac{9}{4}(\tfrac{3}{25}g_1^4 + g_2^4 + \tfrac{2}{5}g_1^2 g_2^2),\end{aligned}$$
$$(4\pi)^2 v^{-1}\gamma_v^{(1)} = -2\textstyle\sum_g(3y_{\mathrm{u}_g}^2 + 3y_{\mathrm{d}_g}^2 + y_{\mathrm{e}_g}^2) + \tfrac{9}{4}(\tfrac{1}{5}g_1^2 + g_2^2).$$

Two-loop contributions

(i) Gauge sector:

$$\begin{aligned}(4\pi)^4 g_1^{-3}\beta_{g_1}^{(2)} = {} & \tfrac{199}{50}g_1^2 + \tfrac{27}{10}g_2^2 + \tfrac{44}{5}g_3^2\\ & - \textstyle\sum_g(\tfrac{17}{5}y_{\mathrm{u}_g}^2 + y_{\mathrm{d}_g}^2 + 3y_{\mathrm{e}_g}^2),\end{aligned}$$
$$(4\pi)^4 g_2^{-3}\beta_{g_2}^{(2)} = \tfrac{9}{10}g_1^2 + \tfrac{35}{6}g_2^2 + 12g_3^2 - \textstyle\sum_g(3y_{\mathrm{u}_g}^2 + 3y_{\mathrm{d}_g}^2 + y_{\mathrm{e}_g}^2),$$
$$(4\pi)^4 g_3^{-3}\beta_{g_3}^{(2)} = \tfrac{11}{10}g_1^2 + \tfrac{9}{2}g_2^2 - 26g_3^2 - 4\textstyle\sum_g(y_{\mathrm{u}_g}^2 + y_{\mathrm{d}_g}^2).$$

(ii) Yukawa sector:

$$\begin{aligned}(4\pi)^4 y_\tau^{-1}\beta_{y_\tau}^{(2)} = {} & 6y_\tau^4 - 9y_\tau^2\textstyle\sum_g(3y_{\mathrm{u}_g}^2 + 3y_{\mathrm{d}_g}^2 + y_{\mathrm{e}_g}^2)\\ & - 9\textstyle\sum_g(3y_{\mathrm{u}_g}^4 + 3y_{\mathrm{u}_g}^4 - \tfrac{2}{3}y_{\mathrm{u}_g}^2 y_{\mathrm{d}_g}^2 + y_{\mathrm{e}_g}^4)\\ & + \tfrac{3}{2}\lambda^2 - 12\lambda y_\tau^2 + (\tfrac{387}{40}g_1^2 + \tfrac{135}{8}g_2^2)y_\tau^2\\ & + 5(\tfrac{17}{20}g_1^2 + \tfrac{9}{4}g_2^2 + 8g_3^2)\textstyle\sum_g y_{\mathrm{u}_g}^2\\ & + 5(\tfrac{1}{4}g_1^2 + \tfrac{9}{4}g_2^2 + 8g_3^2)\textstyle\sum_g y_{\mathrm{d}_g}^2\\ & + \tfrac{15}{4}(g_1^2 + g_2^2)\textstyle\sum_g y_{\mathrm{e}_g}^2 + \tfrac{1371}{200}g_1^4 + \tfrac{27}{20}g_1^2 g_2^2\\ & - \tfrac{23}{4}g_2^4,\end{aligned}$$
$$\begin{aligned}(4\pi)^4 y_{\mathrm{t}}^{-1}\beta_{y_{\mathrm{t}}}^{(2)} = {} & 6y_{\mathrm{t}}^4 - 5y_{\mathrm{t}}^2 y_{\mathrm{b}}^2 + 11y_{\mathrm{b}}^4\\ & + (5y_{\mathrm{b}}^2 - 9y_{\mathrm{t}}^2)\textstyle\sum_g(3y_{\mathrm{u}_g}^2 + 3y_{\mathrm{d}_g}^2 + y_{\mathrm{e}_g}^2)\\ & - 9\textstyle\sum_g(3y_{\mathrm{u}_g}^4 + 3y_{\mathrm{d}_g}^4 - \tfrac{2}{3}y_{\mathrm{u}_g}^2 y_{\mathrm{d}_g}^2 + y_{\mathrm{e}_g}^4)\\ & + \tfrac{3}{2}\lambda^2 - 4\lambda(3y_{\mathrm{t}}^2 + y_{\mathrm{b}}^2) + (\tfrac{223}{40}g_1^2 + \tfrac{135}{8}g_2^2\\ & + 32g_3^2)y_{\mathrm{t}}^2 - (\tfrac{43}{40}g_1^2 - \tfrac{9}{8}g_2^2 + 32g_3^2)y_{\mathrm{b}}^2\\ & + 5(\tfrac{17}{20}g_1^2 + \tfrac{9}{4}g_2^2 + 8g_3^2)\textstyle\sum_g y_{\mathrm{u}_g}^2\\ & + 5(\tfrac{1}{4}g_1^2 + \tfrac{9}{4}g_2^2 + 8g_3^2)\textstyle\sum_g y_{\mathrm{d}_g}^2\\ & + \tfrac{15}{4}(g_1^2 + g_2^2)\textstyle\sum_g y_{\mathrm{e}_g}^2 + \tfrac{1187}{600}g_1^4 - \tfrac{9}{20}g_1^2 g_2^2\\ & - \tfrac{23}{4}g_2^4 + \tfrac{19}{15}g_1^2 g_3^2 + 9g_2^2 g_3^2 - 108g_3^4,\end{aligned}$$
$$\begin{aligned}(4\pi)^4 y_{\mathrm{b}}^{-1}\beta_{y_{\mathrm{b}}}^{(2)} = {} & 6y_{\mathrm{b}}^4 - 5y_{\mathrm{b}}^2 y_{\mathrm{t}}^2 + 11y_{\mathrm{t}}^4\\ & + (5y_{\mathrm{t}}^2 - 9y_{\mathrm{b}}^2)\textstyle\sum_g(3y_{\mathrm{u}_g}^2 + 3y_{\mathrm{d}_g}^2 + y_{\mathrm{e}_g}^2)\\ & - 9\textstyle\sum_g(3y_{\mathrm{u}_g}^4 + 3y_{\mathrm{d}_g}^4 - \tfrac{2}{3}y_{\mathrm{u}_g}^2 y_{\mathrm{d}_g}^2 + y_{\mathrm{e}_g}^4)\\ & + \tfrac{3}{2}\lambda^2 - 4\lambda(3y_{\mathrm{b}}^2 + y_{\mathrm{t}}^2)\\ & + (\tfrac{187}{40}g_1^2 + \tfrac{135}{8}g_2^2 + 32g_3^2)y_{\mathrm{b}}^2\\ & - (\tfrac{79}{40}g_1^2 - \tfrac{9}{8}g_2^2 + 32g_3^2)y_{\mathrm{t}}^2\\ & + 5(\tfrac{17}{20}g_1^2 + \tfrac{9}{4}g_2^2 + 8g_3^2)\textstyle\sum_g y_{\mathrm{u}_g}^2\\ & + 5(\tfrac{1}{4}g_1^2 + \tfrac{9}{4}g_2^2 + 8g_3^2)\textstyle\sum_g y_{\mathrm{d}_g}^2\\ & + \tfrac{15}{4}(g_1^2 + g_2^2)\textstyle\sum_g y_{\mathrm{e}_g}^2 - \tfrac{127}{600}g_1^4 - \tfrac{27}{20}g_1^2 g_2^2\\ & - \tfrac{23}{4}g_2^4 + \tfrac{31}{15}g_1^2 g_3^2 + 9g_2^2 g_3^2 - 108g_3^4.\end{aligned}$$

(iii) Higgs sector:

$$\begin{aligned}(4\pi)^4\beta_\lambda^{(2)} = {} & -78\lambda^3 + 54\lambda^2(\tfrac{1}{5}g_1^2 + g_2^2)\\ & + \lambda(\tfrac{1887}{200}g_1^4 + \tfrac{117}{20}g_1^2 g_2^2 - \tfrac{73}{8}g_2^4)\\ & - \tfrac{3411}{1000}g_1^6 - \tfrac{1677}{200}g_1^4 g_2^2 - \tfrac{289}{40}g_1^2 g_2^4 + \tfrac{305}{8}g_2^6\\ & - 3g_2^4\textstyle\sum_g(3y_{\mathrm{u}_g}^2 + 3y_{\mathrm{d}_g}^2 + y_{\mathrm{e}_g}^2)\\ & - \tfrac{32}{5}g_1^2\textstyle\sum_g(2y_{\mathrm{u}_g}^4 - y_{\mathrm{d}_g}^4 + 3y_{\mathrm{e}_g}^4)\\ & - 256g_3^2\textstyle\sum_g(y_{\mathrm{u}_g}^4 + y_{\mathrm{d}_g}^4)\\ & + 20\lambda\Big((\tfrac{17}{20}g_1^2 + \tfrac{9}{4}g_2^2 + 8g_3^2)\textstyle\sum_g y_{\mathrm{u}_g}^2\\ & + (\tfrac{1}{4}g_1^2 + \tfrac{9}{4}g_2^2 + 8g_3^2)\textstyle\sum_g y_{\mathrm{d}_g}^2\\ & + \tfrac{3}{4}(g_1^2 + g_2^2)\textstyle\sum_g y_{\mathrm{e}_g}^2\Big)\\ & + \tfrac{6}{5}g_1^2\Big((-\tfrac{57}{10}g_1^2 + 21g_2^2)\textstyle\sum_g y_{\mathrm{u}_g}^2\\ & + 3(\tfrac{1}{2}g_1^2 + 3g_2^2)\textstyle\sum_g y_{\mathrm{d}_g}^2\\ & + (-\tfrac{15}{2}g_1^2 + 11g_2^2)\textstyle\sum_g y_{\mathrm{e}_g}^2\Big)\\ & - 48\lambda^2\textstyle\sum_g(3y_{\mathrm{u}_g}^2 + 3y_{\mathrm{d}_g}^2 + y_{\mathrm{e}_g}^2)\\ & - 4\lambda\textstyle\sum_g(3y_{\mathrm{u}_g}^4 + 3y_{\mathrm{d}_g}^4 + y_{\mathrm{e}_g}^4 - 6y_{\mathrm{u}_g}^2 y_{\mathrm{d}_g}^2)\end{aligned}$$

$$
\begin{aligned}
&+ 160 \textstyle\sum_g (3y_{\mathrm{u}_g}^6 + 3y_{\mathrm{d}_g}^6 + y_{\mathrm{e}_g}^6) \\
&- 96 \textstyle\sum_g (y_{\mathrm{u}_g}^4 y_{\mathrm{d}_g}^2 + y_{\mathrm{d}_g}^4 y_{\mathrm{u}_g}^2)\ ,
\end{aligned}
$$

$$
\begin{aligned}
(4\pi)^4 v^{-1}\gamma_v^{(2)} &= -\tfrac{3}{2}\lambda^2 - \tfrac{1293}{800}g_1^4 + \tfrac{271}{32}g_2^4 - \tfrac{27}{80}g_1^2 g_2^2 \\
&- \tfrac{5}{2}(\tfrac{17}{10}g_1^2 + \tfrac{9}{2}g_2^2 + 16g_3^2)\textstyle\sum_g y_{\mathrm{u}_g}^2 \\
&- \tfrac{5}{2}(\tfrac{1}{2}g_1^2 + \tfrac{9}{2}g_2^2 + 16g_3^2)\textstyle\sum_g y_{\mathrm{d}_g}^2 \\
&- \tfrac{15}{4}(g_1^2 + g_2^2)\textstyle\sum_g y_{\mathrm{e}_g}^2 \\
&+ 9\textstyle\sum_g (3y_{\mathrm{u}}^4 + 3y_{\mathrm{d}}^4 - \tfrac{2}{3}y_{\mathrm{u}}^2 y_{\mathrm{d}}^2 + y_{\mathrm{e}}^4).
\end{aligned}
$$

A.2 Neutrino Yukawa contributions to SM β functions

One-loop contributions

(i) Yukawa sector:

$$
\begin{aligned}
&(4\pi)^2 \mathbf{Y}_\nu^{-1}\beta_{\mathbf{Y}_\nu}^{(1)} = \tfrac{3}{2}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu - \mathbf{Y}_{\mathrm{e}}^\dagger\mathbf{Y}_{\mathrm{e}}) + Y_2(S) - \tfrac{9}{20}g_1^2 - \tfrac{9}{4}g_2^2, \\
&(4\pi)^2 \mathbf{Y}_{\mathrm{e}}^{-1}\Delta\beta_{\mathbf{Y}_{\mathrm{e}}}^{(1)} = -\tfrac{3}{2}\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu + \mathrm{Tr}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu), \\
&(4\pi)^2 \mathbf{Y}_{\mathrm{u}}^{-1}\Delta\beta_{\mathbf{Y}_{\mathrm{u}}}^{(1)} = \mathrm{Tr}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu), \\
&(4\pi)^2 \mathbf{Y}_{\mathrm{d}}^{-1}\Delta\beta_{\mathbf{Y}_{\mathrm{d}}}^{(1)} = \mathrm{Tr}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu).
\end{aligned}
$$

(ii) Higgs sector:

$$
\begin{aligned}
&(4\pi)^2 \Delta\beta_\lambda^{(1)} = 4\lambda\mathrm{Tr}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu) - 4\mathrm{Tr}\left((\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^2\right), \\
&(4\pi)^2 v^{-1}\Delta\gamma_v^{(1)} = -\mathrm{Tr}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu).
\end{aligned}
$$

Two-loop contributions

(i) Gauge sector:

$$
\begin{aligned}
&(4\pi)^4 g_1^{-3}\Delta\beta_{g_1}^{(2)} = -\tfrac{3}{10}\mathrm{Tr}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu), \\
&(4\pi)^4 g_2^{-3}\Delta\beta_{g_2}^{(2)} = -\tfrac{1}{2}\mathrm{Tr}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu).
\end{aligned}
$$

(ii) Yukawa sector:

$$
\begin{aligned}
(4\pi)^4 \mathbf{Y}_\nu^{-1}\beta_{\mathbf{Y}_\nu}^{(2)} &= \tfrac{3}{2}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^2 - \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_{\mathrm{e}}^\dagger\mathbf{Y}_{\mathrm{e}} \\
&- \tfrac{1}{4}\mathbf{Y}_{\mathrm{e}}^\dagger\mathbf{Y}_{\mathrm{e}}\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu + \tfrac{11}{4}(\mathbf{Y}_{\mathrm{e}}^\dagger\mathbf{Y}_{\mathrm{e}})^2 \\
&+ Y_2(S)(\tfrac{5}{4}\mathbf{Y}_{\mathrm{e}}^\dagger\mathbf{Y}_{\mathrm{e}} - \tfrac{9}{4}\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu) - \chi_4(S) \\
&+ \tfrac{3}{2}\lambda^2 - 2\lambda(3\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu + \mathbf{Y}_{\mathrm{e}}^\dagger\mathbf{Y}_{\mathrm{e}}) \\
&+ (\tfrac{279}{80}g_1^2 + \tfrac{135}{16}g_2^2)\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu \\
&- (\tfrac{243}{80}g_1^2 - \tfrac{9}{16}g_2^2)\mathbf{Y}_{\mathrm{e}}^\dagger\mathbf{Y}_{\mathrm{e}} + \tfrac{5}{2}Y_4(S) \\
&- (\tfrac{3}{40} - \tfrac{1}{5}n_g)g_1^4 - \tfrac{27}{20}g_1^2 g_2^2 - (\tfrac{35}{4} - n_g)g_2^4, \\
(4\pi)^4 \mathbf{Y}_{\mathrm{e}}^{-1}\Delta\beta_{\mathbf{Y}_{\mathrm{e}}}^{(2)} &= -\mathbf{Y}_{\mathrm{e}}^\dagger\mathbf{Y}_{\mathrm{e}}\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu - \tfrac{1}{4}\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_{\mathrm{e}}^\dagger\mathbf{Y}_{\mathrm{e}} \\
&+ \tfrac{11}{4}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^2 + \tfrac{5}{4}Y_2(S)\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu - 2\lambda\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu \\
&- (\tfrac{3}{16}g_1^2 - \tfrac{129}{16}g_2^2)\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu, \\
(4\pi)^4 \mathbf{Y}_{\mathrm{u}}^{-1}\Delta\beta_{\mathbf{Y}_{\mathrm{u}}}^{(2)} &= (\tfrac{5}{4}\mathbf{Y}_{\mathrm{d}}^\dagger\mathbf{Y}_{\mathrm{d}} - \tfrac{9}{4}\mathbf{Y}_{\mathrm{u}}^\dagger\mathbf{Y}_{\mathrm{u}})\mathrm{Tr}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu) \\
&- \mathrm{Tr}\left(\tfrac{9}{4}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^2 - \tfrac{1}{2}\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_{\mathrm{e}}^\dagger\mathbf{Y}_{\mathrm{e}}\right) \\
&+ \tfrac{15}{8}(\tfrac{1}{5}g_1^2 + g_2^2)\mathrm{Tr}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu), \\
(4\pi)^4 \mathbf{Y}_{\mathrm{d}}^{-1}\Delta\beta_{\mathbf{Y}_{\mathrm{d}}}^{(2)} &= (\tfrac{5}{4}\mathbf{Y}_{\mathrm{u}}^\dagger\mathbf{Y}_{\mathrm{u}} - \tfrac{9}{4}\mathbf{Y}_{\mathrm{d}}^\dagger\mathbf{Y}_{\mathrm{d}})\mathrm{Tr}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu) \\
&- \mathrm{Tr}\left(\tfrac{9}{4}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^2 - \tfrac{1}{2}\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_{\mathrm{e}}^\dagger\mathbf{Y}_{\mathrm{e}}\right) \\
&+ \tfrac{15}{8}(\tfrac{1}{5}g_1^2 + g_2^2)\mathrm{Tr}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu).
\end{aligned}
$$

(iii) Higgs sector:

$$
\begin{aligned}
(4\pi)^4 \Delta\beta_\lambda^{(2)} &= -\tfrac{3}{2}g_2^4\mathrm{Tr}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu) \\
&- \tfrac{3}{10}g_1^2(\tfrac{3}{5}g_1^2 + 2g_2^2)\mathrm{Tr}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu) \\
&+ \tfrac{15}{2}\lambda(\tfrac{1}{5}g_1^2 + g_2^2)\mathrm{Tr}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu) \\
&- 24\lambda^2\mathrm{Tr}(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu) - \lambda\mathrm{Tr}\left((\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^2\right) \\
&+ 2\lambda\mathrm{Tr}\left(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_{\mathrm{e}}^\dagger\mathbf{Y}_{\mathrm{e}}\right) + 20\mathrm{Tr}\left((\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^3\right) \\
&- 4\mathrm{Tr}\left(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu + \mathbf{Y}_{\mathrm{e}}^\dagger\mathbf{Y}_{\mathrm{e}})\mathbf{Y}_{\mathrm{e}}^\dagger\mathbf{Y}_{\mathrm{e}}\right), \\
(4\pi)^2 v^{-1}\Delta\gamma_v^{(2)} &= -\tfrac{15}{8}\mathrm{Tr}\left((\tfrac{1}{5}g_1^2 + g_2^2)\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\right) \\
&+ \tfrac{9}{4}\mathrm{Tr}\left((\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^2\right) - \tfrac{1}{2}\mathrm{Tr}\left(\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_{\mathrm{e}}^\dagger\mathbf{Y}_{\mathrm{e}}\right),
\end{aligned}
$$

where

$$
\begin{aligned}
Y_2(S) &= \mathrm{Tr}(3\mathbf{Y}_{\mathrm{u}}^\dagger\mathbf{Y}_{\mathrm{u}} + 3\mathbf{Y}_{\mathrm{d}}^\dagger\mathbf{Y}_{\mathrm{d}} + \mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu + \mathbf{Y}_{\mathrm{e}}^\dagger\mathbf{Y}_{\mathrm{e}}), \\
\chi_4(S) &= \tfrac{9}{4}\mathrm{Tr}\left(3(\mathbf{Y}_{\mathrm{u}}^\dagger\mathbf{Y}_{\mathrm{u}})^2 + 3(\mathbf{Y}_{\mathrm{d}}^\dagger\mathbf{Y}_{\mathrm{d}})^2 + (\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu)^2\right. \\
&\left.+ (\mathbf{Y}_{\mathrm{e}}^\dagger\mathbf{Y}_{\mathrm{e}})^2 - \tfrac{2}{3}\mathbf{Y}_{\mathrm{u}}^\dagger\mathbf{Y}_{\mathrm{u}}\mathbf{Y}_{\mathrm{d}}^\dagger\mathbf{Y}_{\mathrm{d}} - \tfrac{2}{9}\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu\mathbf{Y}_{\mathrm{e}}^\dagger\mathbf{Y}_{\mathrm{e}}\right), \\
Y_4(S) &= \mathrm{Tr}\left((\tfrac{17}{20}g_1^2 + \tfrac{9}{4}g_2^2 + 8g_3^2)\mathbf{Y}_{\mathrm{u}}^\dagger\mathbf{Y}_{\mathrm{u}}\right. \\
&+ (\tfrac{1}{4}g_1^2 + \tfrac{9}{4}g_2^2 + 8g_3^2)\mathbf{Y}_{\mathrm{d}}^\dagger\mathbf{Y}_{\mathrm{d}} \\
&\left.+ \tfrac{3}{4}(\tfrac{1}{5}g_1^2 + g_2^2)\mathbf{Y}_\nu^\dagger\mathbf{Y}_\nu + \tfrac{3}{4}(g_1^2 + g_2^2)\mathbf{Y}_{\mathrm{e}}^\dagger\mathbf{Y}_{\mathrm{e}}\right).
\end{aligned}
$$

Our definition of the Yukawa matrices and the invariants immediately generalizes that of [14].

A.3 Heavy neutrino contributions to SM β functions

For simplicity we put here $\nu_4 = N, e_4 = E$. In what follows $n_{\mathrm{g}} = 4$ and the generation index g runs over the values $g = 1, \ldots, 4$. Our normalization for the Yukawa couplings y_{f_g} corresponds to $(\mathbf{Y}_f^{\mathrm{diag}})_{gg'} = \sqrt{2}y_{f_g}\delta_{gg'}$. In practice, one has $y_{\nu_g} = 0$ for $g \neq 4$.

One-loop contributions

(i) Yukawa sector:

$$
\begin{aligned}
&(4\pi)^2 y_N^{-1}\beta_{y_N}^{(1)} = 3y_N^2 - 3y_E^2 + Y_2(S) - \tfrac{9}{20}g_1^2 - \tfrac{9}{4}g_2^2, \\
&(4\pi)^2 y_E^{-1}\Delta\beta_{y_E}^{(1)} = -y_N^2, \\
&(4\pi)^2 y_f^{-1}\Delta\beta_{y_f}^{(1)} = 2y_N^2,
\end{aligned}
$$

where $f \neq N, E$.

(ii) Higgs sector:

$$
\begin{aligned}
&(4\pi)^2 \Delta\beta_\lambda^{(1)} = 8\lambda y_N^2 - 16y_N^4. \\
&(4\pi)^2 v^{-1}\Delta\gamma_v^{(1)} = -2y_N^2.
\end{aligned}
$$

Two-loop contributions

(i) Gauge sector:

$$
\begin{aligned}
&(4\pi)^4 g_1^{-3}\Delta\beta_{g_1}^{(2)} = -\tfrac{3}{5}y_N^2, \\
&(4\pi)^4 g_2^{-3}\Delta\beta_{g_2}^{(2)} = -y_N^2.
\end{aligned}
$$

(ii) Yukawa sector:

$$
\begin{aligned}
(4\pi)^4 y_N^{-1}\beta_{y_N}^{(2)} &= 6y_N^4 - 5y_E^2 y_N^2 + 11y_E^4 + \tfrac{1}{2}Y_2(S)(5y_E^2 - 9y_N^2) \\
&- \chi_4(S) + \tfrac{3}{2}\lambda^2 - 4\lambda(3y_N^2 + y_E^2) \\
&+ (\tfrac{279}{40}g_1^2 + \tfrac{135}{8}g_2^2)y_N^2 - (\tfrac{243}{40}g_1^2 - \tfrac{9}{8}g_2^2)y_E^2 \\
&+ \tfrac{5}{2}Y_4(S) + \tfrac{3}{5}(-\tfrac{1}{8} + \tfrac{1}{3}n_g)g_1^4 - \tfrac{27}{20}g_1^2 g_2^2
\end{aligned}
$$

$$- (\tfrac{35}{4} - n_g) g_2^4,$$

$$(4\pi)^4 y_E^{-1} \Delta\beta_{y_E}^{(2)} = 11 y_N^4 - 5 y_E^2 y_N^2 + \tfrac{5}{2} Y_2(S) y_N^2 - 4\lambda y_N^2 - (\tfrac{3}{8} g_1^2 - \tfrac{129}{8} g_2^2) y_N^2,$$

$$(4\pi)^4 y_{\mathrm{u}}^{-1} \Delta\beta_{y_{\mathrm{u}}}^{(2)} = (5 y_{\mathrm{d}}^2 - 9 y_{\mathrm{u}}^2) y_N^2 - (9 y_N^2 - 2 y_E^2) y_N^2 + \tfrac{15}{4} (\tfrac{1}{5} g_1^2 + g_2^2) y_N^2,$$

$$(5 y_{\mathrm{u}}^2 - 9 y_{\mathrm{d}}^2) y_N^2 - (9 y_N^2 - 2 y_E^2) y_N^2 + \tfrac{15}{4} (\tfrac{1}{5} g_1^2 + g_2^2) y_N^2.$$

(*iii*) Higgs sector:

$$(4\pi)^4 \Delta\beta_\lambda^{(2)} = -3 g_2^4 y_N^2 - \tfrac{3}{5} g_1^2 (\tfrac{3}{5} g_1^2 + 2 g_2^2) y_N^2 + 15\lambda (\tfrac{1}{5} g_1^2 + g_2^2) y_N^2 - 48\lambda^2 y_N^2 - 4\lambda y_N^4 + 8\lambda y_E^2 y_N^2 + 160 y_N^6 - 32 y_E^2 (y_N^2 + y_E^2) y_N^2,$$

$$(4\pi)^2 v^{-1} \Delta\gamma_v^{(2)} = -\tfrac{15}{4} (\tfrac{1}{5} g_1^2 + g_2^2) y_N^2 + 9 y_N^4 - 2 y_N^2 y_E^2,$$

where

$$Y_2(S) = 2 \textstyle\sum_g (3 y_{\mathrm{u}_g}^2 + 3 y_{\mathrm{d}_g}^2 + y_{\nu_g}^2 + y_{\mathrm{e}_g}^2),$$

$$\chi_4(S) = 9 \textstyle\sum_g (3 y_{\mathrm{u}_g}^4 + 3 y_{\mathrm{d}_g}^4 - \tfrac{2}{3} y_{\mathrm{u}_g}^2 y_{\mathrm{d}_g}^2 + y_{\nu_g}^4 + y_{\mathrm{e}_g}^4 - \tfrac{2}{9} y_{\nu_g}^2 y_{\mathrm{e}_g}^2),$$

$$Y_4(S) = 2 \textstyle\sum_g \Big((\tfrac{17}{20} g_1^2 + \tfrac{9}{4} g_2^2 + 8 g_3^2) y_{\mathrm{u}_g}^2 + (\tfrac{1}{4} g_1^2 + \tfrac{9}{4} g_2^2 + 8 g_3^2) y_{\mathrm{d}_g}^2 + \tfrac{3}{4} (\tfrac{1}{5} g_1^2 + g_2^2) y_{\nu_g}^2 + \tfrac{3}{4} (g_1^2 + g_2^2) y_{\mathrm{e}_g}^2 \Big).$$

In fact, only the third and fourth generations contribute here in the sums.

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Note added in proof

More conservative recent restrictions $M_{\mathrm{H}} \leq 262$ GeV or $M_{\mathrm{H}} \leq 300$ GeV from the papers F. Teubert, hep-ph/9811414 and G. D'Agostini, G. Degrassi, hep-ph/9902226, respectively, though render our conclusion about non-existence of the fourth heavy chiral family less reliable, nevertheless do not contradict it.